

Dynamic analysis of a reinforced concrete shear wall with strain rate effect

Synopsis

A simplified analysis method for a reinforced concrete shear wall structure considering strain rate effects is presented. The model uses a strut-and-tie simplified model with concrete strut and steel rebar elements comprising a three-element Maxwell model. The results of dynamic response analysis show very good agreement with the test results of the ISP (International Standard Problem sponsored by OECD/NEA/CSNI) shear wall in the nonlinear response region including at the maximum strength point. Some differences are observed in the elastic range and failure mode.

The effective strain rate for increasing strength is less than one half of the maximum strain rate and 70% of the root mean value in this study.

Keywords: strain rate effect; shear wall; simplified model; dynamic analysis; Maxwell model; reinforced concrete

Introduction

Since the mechanical properties of concrete and steel are functions of loading speed, a dynamic analysis needs to consider the strain rate effect. Many tests were carried out to investigate the strain rate effects of reinforced concrete structural members and materials. Recently, the design strength of materials depending on the strain rate has been proposed¹⁾²⁾. Fu et al.³⁾ reviewed the effects of loading rate on reinforced concrete and showed a stress-strain relation model that considers the strain rate effect.

However, investigations on dynamic response analysis that considers the strain rate effects have seldom been reported, because it is difficult to make the analytical model and to perform the mathematical calculations. Some papers have been reported. Kitagawa et al.⁴⁾ used a three-element Maxwell model consisted of two spring elements and one damper element for reinforced concrete members and got a good agreement between tests and analysis with consideration of strain rate and stress relaxation. Fujimoto et al.⁵⁾ proposed an analytical model using the three-element Maxwell model for concrete and reinforcing bars and performed dynamic response analysis of a frame structure, concluding that the model enabled earthquake response analysis considering the strain rate effects.

The strain rate effect for a frame structure during a severe earthquake is reported to be not so significant, and can be ignored for the dynamic response analysis. The reasons are as follows. The behavior is mainly affected by yielding of the main rebars of beams. The equivalent natural period at which the rebars of beams just yield is more than one second. When a building vibrates according to a sine wave with a period of one second and the strain amplitude of the rebars is about 0.002 (yielding), the maximum strain rate of the rebars is about 0.01/s. At this rate, the strain rate effect of rebars is not so significant.

For wall type structures such as a reactor building, however, the behavior during a severe earthquake is affected by concrete components, and the strain rate effect is not negligible. For example, when a structure vibrates according to a sine wave with a period of 0.3 second and the strain amplitude is 0.0015 (near the maximum strength of concrete), the maximum strain rate of concrete is about 0.03/s. At this rate, concrete strength is increased about 1.2 times based on previous studies⁵⁾.

For model tests, furthermore, the strain rate effect is very significant because the natural frequency is very high based on the dimensional requirements. Recently, the shear wall vibration table test results were opened to the public as an International Standard Problem (ISP) sponsored by OECD/NEA/CSNI⁶⁾. This paper investigates the strain rate effect on the dynamic response analysis of the shear wall test specimen using the three-element Maxwell model.

RESEARCH SIGNIFICANCE

For wall type structures, the strain rate effect is not negligible. This paper showed the analytical model to investigate the strain rate effect in the simulation of the dynamic response for a shear wall test specimen. The dynamic analysis was carried out using a simplified arch-strut model with a three-element Maxwell model for

each element, and good agreement was obtained between tested and calculated results. The effective strain rate is discussed for a rough estimate of strain rate effect.

Summary of the ISP test

The test was carried out by the Nuclear Power Engineering Corporation entrusted by the Ministry of International Trade & Industry of the Japanese government. The data of the test results is provided as a Seismic Shear Wall ISP sponsored by OECD/NEA/CSNI.

Fig. 1 shows the specimen tested. The web wall is 75 mm thick, 2900 mm long clear span, 2020 mm clear height with a shear span ratio of 0.8. The flange walls are 100 mm thick and 2980 mm long. Deformed bar of nominal diameter 6.35 mm with spacing pitch 70 mm in a double layer is used for the vertical and horizontal reinforcement of the web wall. Total mass including top slab is 122,000 kg. Vertical compressive stress in the wall is 1.5 MPa. The specimen is subjected to excitations in one direction. The vibration test steps are set corresponding to the five target response levels. Each level uses the same input acceleration waveforms with varying amplitude. Fig. 2 shows the input acceleration waveform for Run-4.

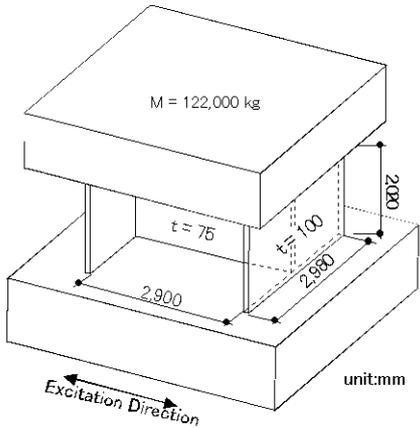


Fig. 1 Test specimen

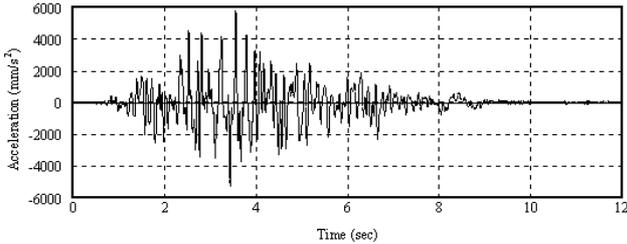


Fig. 2 Input acceleration time history for run-4

In this study, the following three steps are investigated :

- i) Small amplitude level in elastic range (Run-1)
 - ii) Shear deformation angle of about 2/1000 rad (Run-4)
 - iii) Shear deformation angle of about 4/1000 rad (Run-5)
- (Run-2 and Run-3 are eliminated in this study)

Before each test, small amplitude vibration tests are carried out to obtain the dynamic characteristics of the specimen. The natural frequency and damping factor obtained by these tests are summarized in Table 1.

Table 1 Dynamic characteristics of specimen obtained by small amplitude vibration test

	Natural Frequency (Hz)	Damping Factor (%)
Before Run-1	13.2	1.1
Before Run-3	11.3	2.5
Before Run-4	9.0	3.0
Before Run-5	7.7	4.0

Analytical method

ANALYTICAL MODEL

A strut-and-tie simplified model is used for the analysis. The reasons for using this model are as follows :

- i) The shear span ratio is small, and the arch strut is the key member.
- ii) As the vertical stress is small, the principal stress of the web concrete is almost equal to 45 degrees.
- iii)

This model can be applied easily to simulate the response of this type of shear wall. Fig. 3 shows the

strut-and-tie model used in this study. Sections of concrete strut for web are determined to be equal to the shear stiffness of the web. The other members' sections are equal to the full section of each divided area. For the rebars, pull-out extension from the basement is considered as a spring.

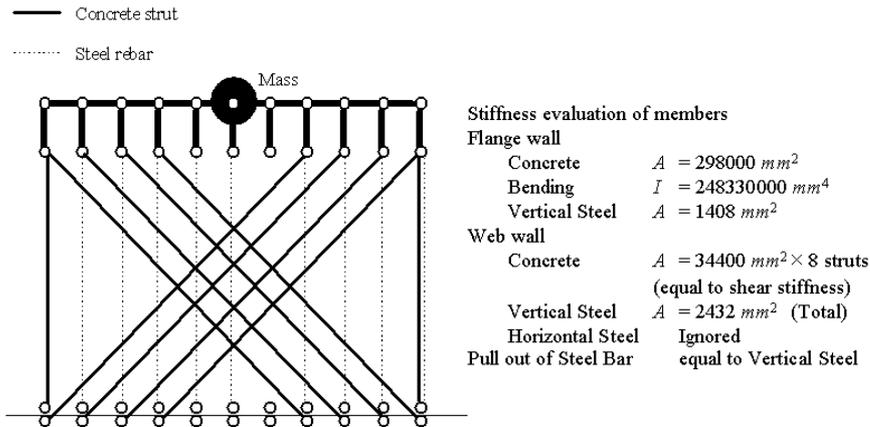


Fig. 3 Truss model scheme

Each truss element of the concrete struts and steel rebars is modeled by the three-element Maxwell model as shown in Fig. 4. The Maxwell model is used to simulate the dynamic visco-elastic properties for the material model. E_0 is the stiffness of the main spring. The properties of the main spring are determined by static material tests. The other two elements of the Maxwell model are viscosity coefficient η and stiffness E_1 , and the properties of this part are determined by dynamic material tests with a variety of loading rates. The tangent stiffness of E_1 is assumed to be proportional to E_0 . This assumption can consider the nonlinear properties of the material.

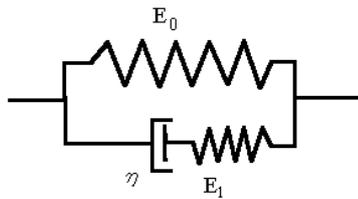


Fig. 4 Three elements Maxwell model

CONCRETE STRUT

A multi-linear stress-strain relationship as shown in Fig. 5 is used for the concrete strut. Compression strength is reduced 0.55 times of the reported cylinder compression strength as cracked concrete. Tension strength is reduced 0.6 times of the splitting tension strength of a cylinder. These reduction values are determined from the literature^{(6) (7) (8)} considering the proportions of the concrete strut. This relation dose not include the strain rate effect. Hysteresis rules are a simple origin oriented model with additional slip phenomenon.

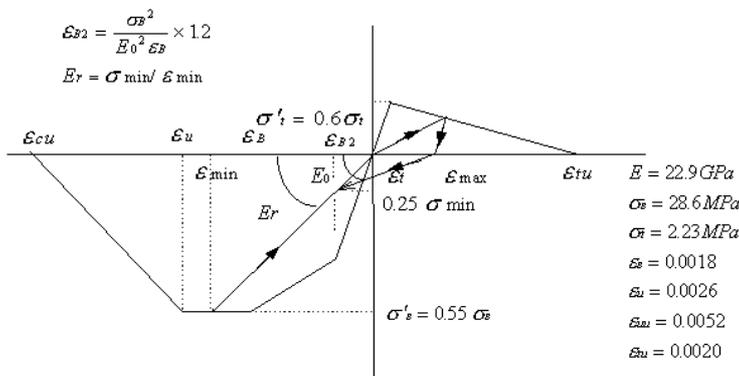


Fig. 5 Stress-Strain relationship of concrete strut

Fujimoto et al.⁽⁵⁾ showed that the values of $E_1=0.25E_0$ and $\eta = 400 \text{ MPa}\cdot\text{s}$ for the three-element Maxwell

model can simulate the strain rate effect of concrete with good agreement. Stress-strain relationships of this model for monotonic loading with various strain rates are shown in Fig. 6. This figure shows that both stiffness and compressive strength increase with increasing strain rate, as indicated in the literature³⁾. Fig. 7 shows the relationship between strain rate and strength ratio (dynamic strength/static strength) for test results⁵⁾ and for the model. The agreement is good.

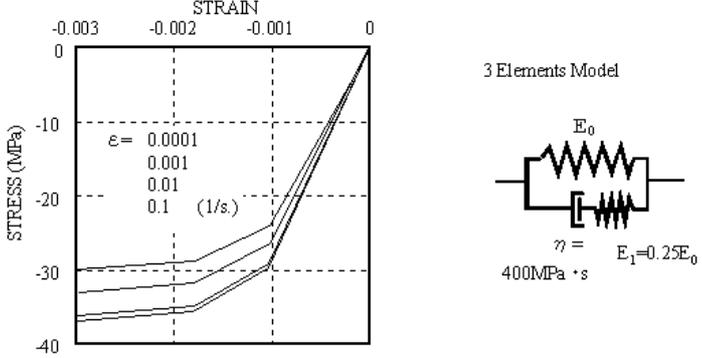


Fig. 6 Stress-Strain relationship of concrete strut with various strain rate

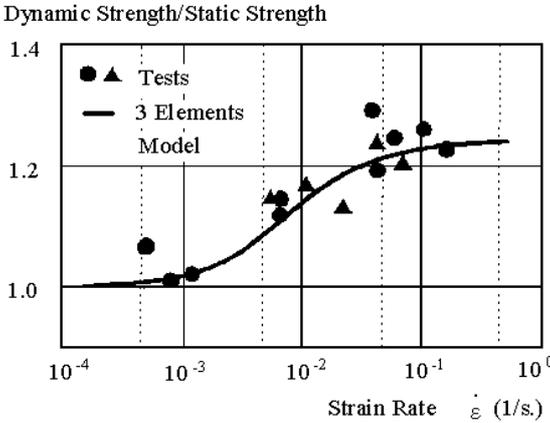


Fig. 7 Strain rate effect of concrete (according to Ref. 5)

STEEL BAR

The simple bilinear model shown in Fig. 8 is used for the steel rebars. Young’s modulus and yield strength are reported values. At the bottom of the vertical rebar, a spring element is placed to consider pull-out extension from the basement. The hysteresis relation of the spring is shown in Fig. 9.

To consider the strain rate effect, the values of $E_1=0.30E_0$ and $\eta = 1000 \text{ MPa}\cdot\text{s}$ for the three-element Maxwell model are used. Stress-strain relationships for monotonic loading of this model with various strain rates are shown in Fig. 10. The modulus of elasticity of this model changes slightly with increasing strain rate. In the test results, the modulus of elasticity remains unchanged³⁾. This is a small problem of the three-element Maxwell model for a steel rebar. This is not so significant for the wall type specimen because the behavior of the concrete dominates the efficiency. Fig. 11 shows good agreement between the test results and the model in the relationship between strain rate and strength ratio.

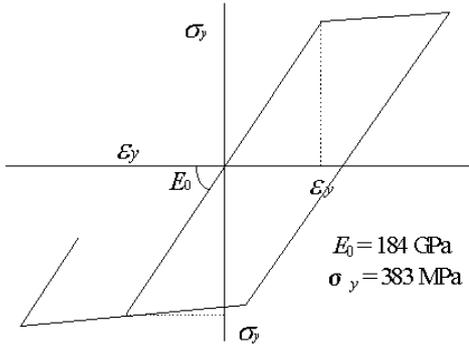


Fig. 8 Stress-Strain relationship of concrete strut

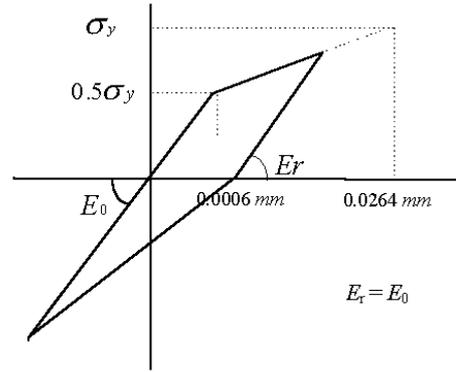


Fig. 9 Hysteresis loop of pull-out members

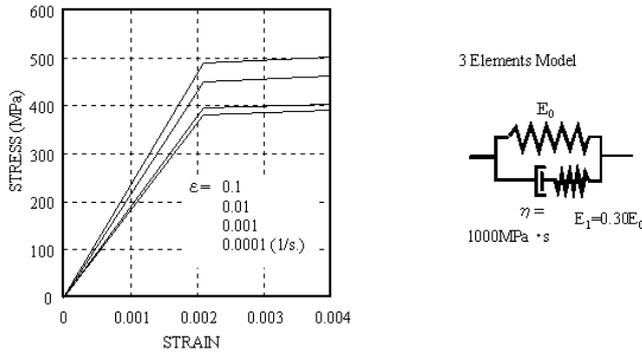


Fig. 10 Stress-Strain relationship of rebar with various strain rate

Dynamic Strength/Static Strength

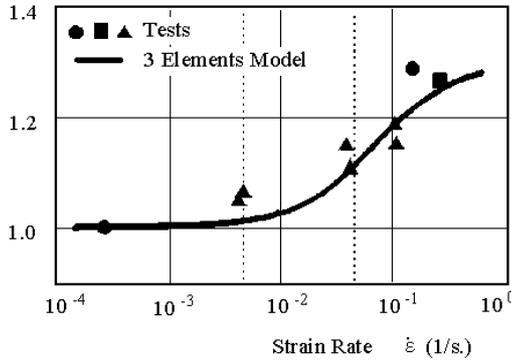


Fig. 11 Strain rate effect of steel rebar (according to Ref. 5)

EVALUATION OF DAMPING

The complex stiffness of the three-element Maxwell model is given by Eq. 1. Here, T_1 is relaxation time and is given by the equation $T_1 = \eta / E_1$.

$$E = E_0 + E_1 \frac{\omega^2 T_1^2}{1 + \omega^2 T_1^2} + iE_1 \frac{\omega T_1}{1 + \omega^2 T_1^2} \dots\dots\dots(1)$$

Complex damping coefficient β is thus given by

$$\beta = \frac{E_1 \omega T_1}{2\{E_0 + (E_0 + E_1)\omega^2 T_1^2\}} \dots\dots\dots(2)$$

The dynamic characteristics in the elastic range are mainly affected by the concrete properties. Substituting the material properties of concrete ($E_1 = 0.25E_0$, $T_1 = \eta / 0.25E_0$) and frequency of 13.2Hz into Eq. 2, a complex damping factor of 1.7% is obtained. This value is larger than the reported value obtained by small amplitude vibration tests before Run-1 shown in Table 1. Table 2 summarizes the estimated damping

factors for the three-element Maxwell model. In this table, the equivalent stiffness of concrete is assumed to be proportional to the square of the natural frequency. The damping factors in Table 2 are smaller than the values in Table 1 except the value before Run-1. The additional external damping may be needed to get good agreement between test and analysis.

CALCULATION METHOD

The mathematical solution of the Maxwell model in time domain is a function of e (natural base of logarithm). This was difficult to calculate in the dynamic analysis. Recently, Hatada et al.¹⁰⁾ gave a step by step solution of this model. This paper applies this solution to the problem with varying stiffness. Fig. 12 shows the three-element model and the symbols of properties. The element displacement and internal force are defined as the difference of both end nodes :

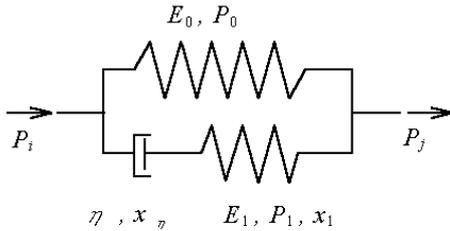


Fig. 12 Three elements model and symbols of properties

$$x = x_i(t) - x_j(t)$$

$$P = P_i(t) - P_j(t) \dots \dots \dots (3)$$

Assuming internal force P_1 and stiffness E_1 in the Maxwell model are constant during Δt ,

$$P_1(t) = \eta \cdot \dot{x}_\eta(t) = \eta \{ \dot{x}(t) - \dot{x}_1(t) \} = \eta \cdot \dot{x}(t) - \frac{\eta}{E_1} \dot{P}_1(t) \dots \dots \dots (4)$$

Internal force ${}_n P_1$ at step n and internal force ${}_{n+1} P_1$ at step $n+1$ are

$${}_n P_1 = \eta \cdot \dot{x} - \frac{\eta}{E_1} \dot{P}_1$$

$${}_{n+1} P_1 = \eta \cdot \dot{x} - \frac{\eta}{E_1} \dot{P}_1 \dots \dots \dots (5)$$

It is assumed that P changes linearly,

$${}_{n+1} P_1 = {}_n P_1 + \frac{\Delta t}{2} ({}_{n+1} \dot{P}_1 + {}_n \dot{P}_1) \dots \dots \dots (6)$$

Substituting Eq. 6 into Eq. 5, following equation is obtained :

$${}_{n+1} P_1 = \frac{\Delta t \cdot \eta \cdot E_1}{\Delta t \cdot E_1 + 2\eta} \dot{x} + \frac{\Delta t \cdot \eta \cdot E_1}{\Delta t \cdot E_1 + 2\eta} \dot{x} + \frac{2\eta - \Delta t \cdot E_1}{\Delta t \cdot E_1 + 2\eta} P_1 \dots \dots \dots (7)$$

From this equation, the force of the Maxwell model can be calculated by the velocity of the previous and present step and the force of the previous step. A small time step increment is required for calculation. Hatada et al.¹⁰⁾ confirmed the numerical stability and accuracy of this solution method. Good results can be obtained for the limitation that $\omega_0 \cdot \Delta t$ is less than 0.1. Here, ω_0 is natural circular frequency.

Analytical Result

In this study, dynamic freedom is reduced to one in the horizontal direction for the mass. The time interval is set at 0.0001 second. In this time step, stiffness is assumed to be constant, and unbalanced force is corrected in the next step. Damping is assumed to be the sum of the external damping in proportion to the mass and the complex damping of the three-element Maxwell model of concrete arch struts.

RUN-1

The displacement response time history of analytical and tested results are compared in Fig. 13. The damping coefficient of the external damping is assumed to be zero because the estimated complex damping of

the three-element Maxwell model is larger than the reported value obtained by the small amplitude vibration test. The displacement response analysis can not simulate the test results after four seconds. The analytical result converges after four seconds, even though the test specimen continues to vibrate with a large amplitude. The main reason for this difference seems to be over-estimation of damping in the analytical model.

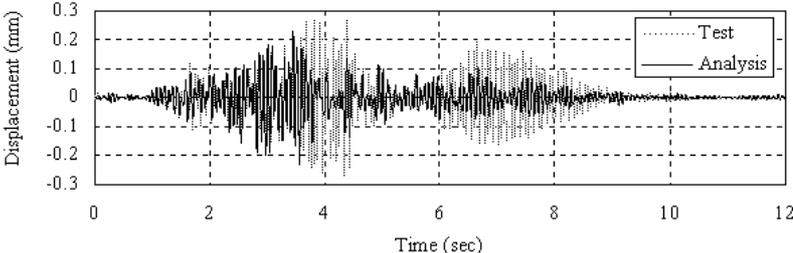


Fig. 13 Time history of displacement response for Run-1

To confirm this, a comparison of the analytical result with the model of no strain rate effect (E_1 of the three-element Maxwell model is zero) with damping of 1.1% is shown in Fig. 14. Good agreement is shown. This is just elastic response. In the elastic range, the strain rate effect is not significant. The Maxwell model shows not only a strain rate effect but also over-estimation of damping. Accordingly, the analytical result becomes smaller than the test result.

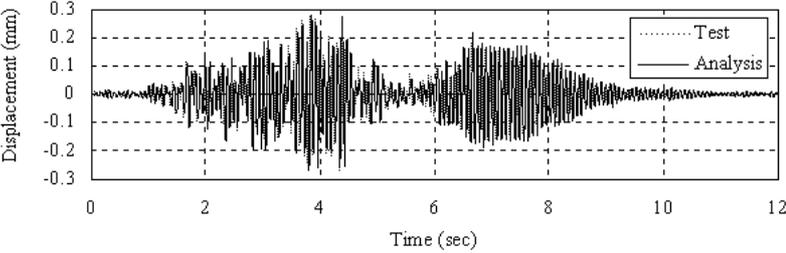


Fig. 14 Time history of displacement response without the strain rate effect for Run-1

RUN-4

The damping factor obtained by the small amplitude vibration test before run-4 is 3%. The complex damping factor of the concrete strut using an equivalent stiffness is 1.2%, so an external damping of 2% is added for the analysis. Fig. 15 compares the displacement response time history for the analytical and tested result. Good agreement is shown. The internal force-displacement relationship is shown in Fig. 16, which shows a small difference. The main difference is the unloading stiffness from the virgin loop. The stiffness of the analytical model is larger than the stiffness of the test result, probably due to the difference of modeling of concrete strut properties. Other difference is unshapely hysteresis loop in the test result compared with the analytical result. The analytical model has only a single dynamic freedom in the horizontal direction and cannot consider second mode vibration. The test result includes the second mode vibration caused by the rotational moment of inertia of the top mass.

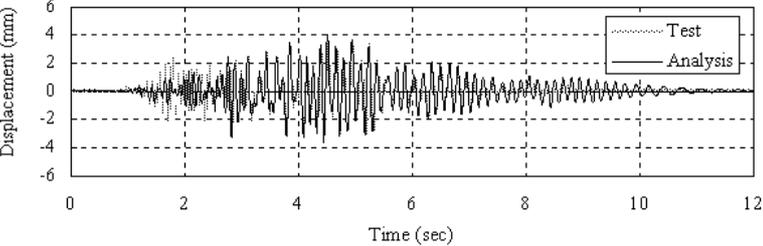


Fig. 15 Time history of displacement response for Run-4

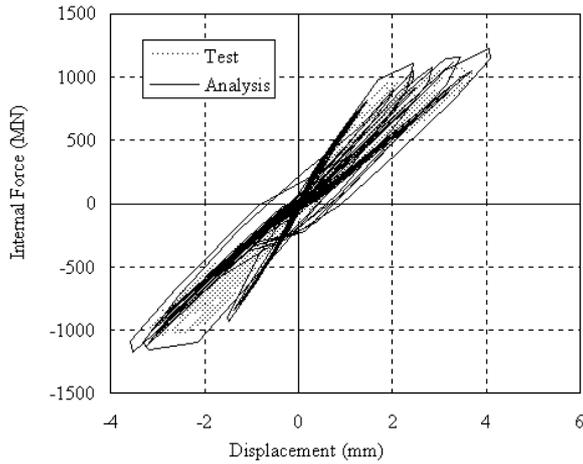


Fig. 16 Internal force-displacement relationship for Run-4

The result of comparison analysis of the model without the strain rate effect is shown in Figs. 17 and 18 as time history and internal force-displacement relationships. The damping is assumed to be 3% according to the small amplitude vibration test result. The analytical result shows a larger response than the test result. This means that for the dynamic response analysis to be accurate, both damping and strain rate effect must be assumed correctly.

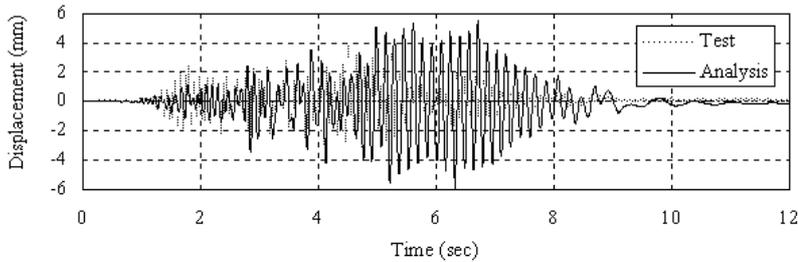


Fig. 17 Time history of displacement response without the strain rate effect for Run-4

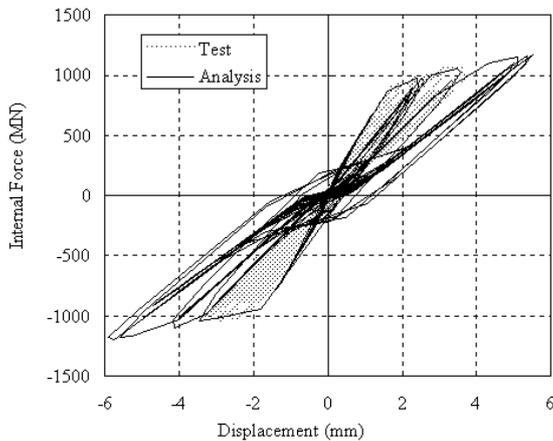


Fig. 18 Internal force-displacement relationship without the strain rate effect for Run-4

Fig. 19 shows the time history of strain and strain rate of an arch concrete strut. The maximum strain rate is around 0.03/s to 0.04/s. At this strain rate, the strength increase factor is 1.2 as shown in Fig. 7. The root mean square value of strain rate as an effective value between 4 to 6 second is about 0.02/s. At this rate, the strength increase factor is about 1.17. Fig. 20 shows the stress-strain relationship of the concrete arch strut. The Maxwell stress is about 0.15 times the main spring stress and total stress is about 1.15 times, not 1.2 times, because the maximum speed will occur near zero displacement, and at the maximum displacement, the speed reduces nearly to zero. The strain rate for the strength ratio of 1.15 reckoned backward from Fig. 7 is about 0.013/s. Accordingly, the effective strain rate for strength increase is less than one half of the maximum strain rate and 70% of the root mean square value.

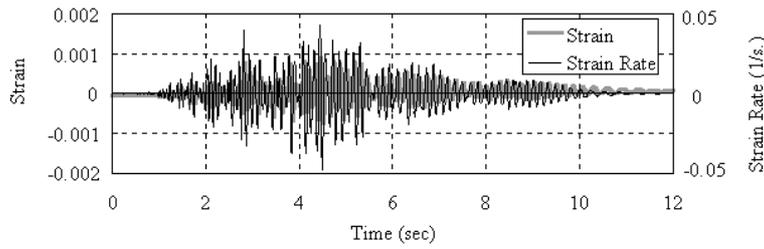


Fig. 19 Strain and strain rate time history of the arch concrete strut

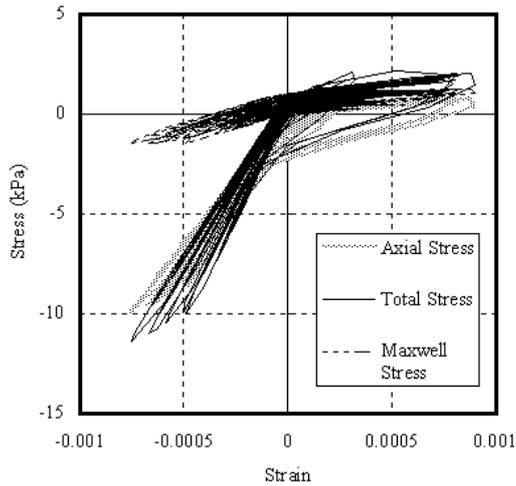


Fig. 20 Stress-Strain relationship of concrete strut as three element Maxwell model

RUN-5

The damping factor of the test before run-5 is 4% and the complex damping factor of the three-element Maxwell model for the concrete strut is 1.0%, so external damping of 3% is added for the analysis. Fig. 21 compares the displacement response time history for the analytical and test results. The flat waveform of the test result is due to ceiling by the over range of the measurement. Good agreement is shown before failure at four seconds. The internal force-displacement relationship is shown in Fig. 22. The test specimen failed in shear slip mode. The analytical model has only arch struts and tie rebars for the web member, and can not simulate this type of failure mode. Regarding the shear slip failure mode, if the system fails in one side, the strength of the other side also disappears. In the arch-strut model, the strength of the opposite side is independent. This difference is clear in the positive side response after failure in the negative side. An unshapely hysteresis loop in the test result compared with the analytical result is shown, the same as in Run-4 caused by the rotational moment of inertia of top mass.

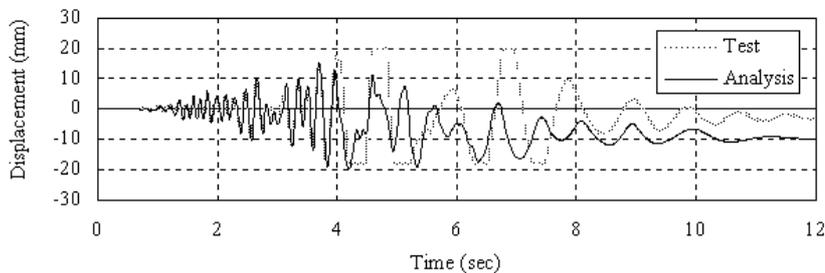


Fig. 21 Time history of displacement response for Run-5

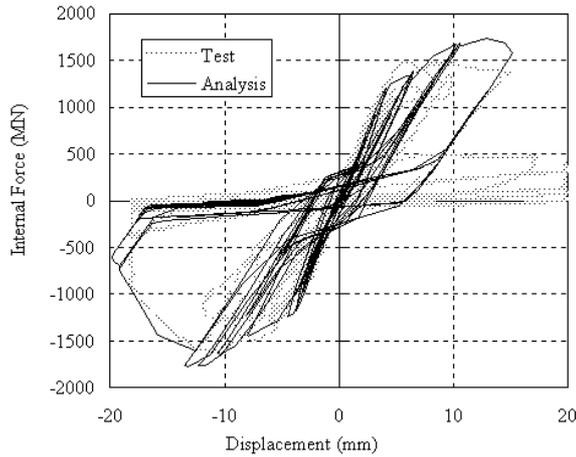


Fig. 22 Internal force-displacement relationship for Run-5

Fig. 23 shows time history of strain and strain rate of an arch concrete strut. The maximum strain rate is about 0.05/s around 2 seconds at which the stress becomes the maximum. At this strain rate, the strength increase factor is about 1.22 as shown in Fig. 7. The root mean square value around 2 seconds is about 0.03/s, and the strength increase factor is about 1.20. Fig. 24 shows the stress-strain relationship of the concrete arch strut. The total stress is about 1.17 times the main spring stress. The strain rate for the strength ratio of 1.17 reckoned backward from Fig. 7 is about 0.02/s. Here again, the effective strain rate for strength increase is less than one half of the maximum strain rate and 70% of the root mean square value.

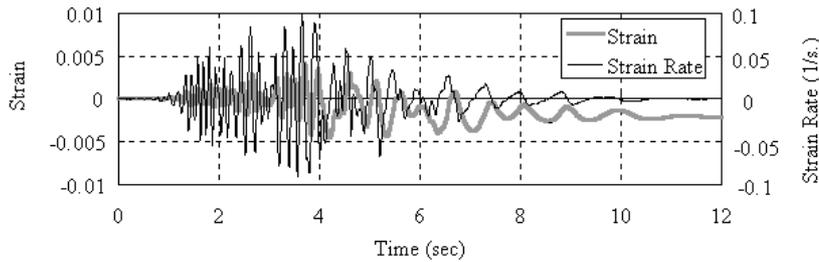


Fig. 23 Strain rate time history of the arch concrete strut

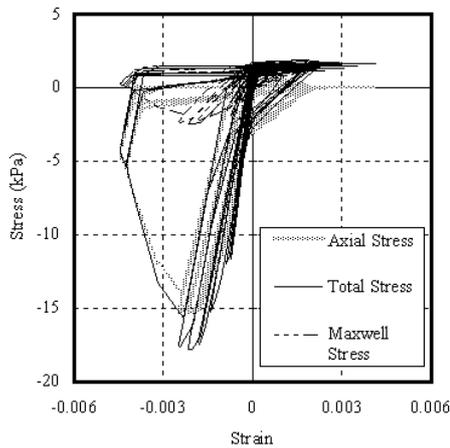


Fig. 24 Stress-Strain relationship of concrete strut as three element Maxwell model

In the Fig. 22, the stress-strain relationship on the tension side cannot be overlooked. The Maxwell stress and the total stress are constant values. The value becomes minus by unloading, and remains constant because of the zero stiffness after tension failure. The problem seems to be in the assumption of the stress-strain relationship of the concrete strut. However, we have no information for the strain rate effect on the tension stiffening relationship. To confirm this, plane shear tests at various loading speeds must be performed.

Conclusions

This paper investigated the strain rate effect in the simulation of the dynamic response for a shear wall test specimen. The dynamic analysis was carried out using a simplified arch-strut model with a three-element Maxwell model for each element. The following results were obtained :

1. The simplified arch-strut model with the three-element Maxwell model can simulate the dynamic behavior of a shear wall very well.
2. The three-element Maxwell model has some damping, the amount of which is evaluated by complex damping. If a system has little damping, simulation using the three-element Maxwell model will over-estimate the damping effect.
3. The effective strain rate for the strength increase is less than one half of the maximum strain rate and 70% of the root mean square value in this study.
4. The arch-strut model cannot simulate shear slip failure. In shear slip mode, when the system fails in one side, the strength of the other side also disappears. In the arch-strut model, the strength depends on each arch concrete strut independently.

The analysis procedure described in this paper produces results that agree well with the test results. However, the strain rate effect in the arch strut is still not clear. Plane shear tests with various loading speeds need to be conducted in the future.

Acknowledgments

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