ESTIMATION OF DISPLACEMENT RESPONSES FOR R/C BUILDINGS USING THE RESPONSE SPECTRUM

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ABSTRACT

This paper proposes a method to estimate the maximum displacement response of reinforced concrete buildings in an earthquake using the response spectrum. Nonlinear earthquake response calculations of 12,600 systems were carried out for 20 ground motions. The results were classified by using the dimensionless parameters based on properties of the system and the frequency characteristics of the ground motion, and the relation of the dimensionless parameters, strength ratio *SR* and displacement ratio *DR* was plotted. This relation was fitted by a hyperbolic curve for each plotted figure and the estimation equation (7) was proposed. This estimation showed satisfactory results and were on the safe side.

INTRODUCTION

The structural performance must be clarified in earthquake-resistant design, when the emphasis is on the performance. It has been shown that the displacement response is the simple index of performance. Many works to estimate the displacement response have been carried out. They were summarized in references [1,2]. The authors [3,4] have examined design methods based on response displacement, and investigated the condition of equal displacement response in elasto-plasticity displacement response of the one-mass system using the bi-linear hysteresis model.

In those studies, the elastic response spectrum was smoothed as shown in Figure 1 for each ground motion used in the calculation. The structures were idealized as SDOF systems having bi-linear hysteresis. Many earthquake response analyses were conducted for systems using dimensionless parameters *TR* and *SR*. *TR* is the ratio of the initial period in the system to the boundary period T_1 between the constant acceleration spectrum region and the constant velocity spectrum region as shown in Figure 1. *SR* is the ratio of the shear strength of the system to the elastic resistant response.

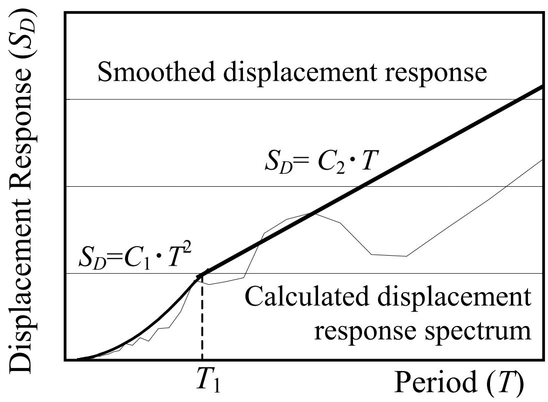


Figure 1: Smoothed displacement response spectrum

The calculated displacement response was normalized as *DR* divided by the smoothed elastic displacement response spectral value with the same initial period and damping factor. The results are plotted in Figure 2 as the relation of *SR* and *TR*, and can be summarized as follows.

1) *SR*+*TR*≧1, The displacement response does not exceed the elastic displacement response regardless of the type of hysteresis model and strength.
2) *SR*+*TR*<1, *DR* is proportional to *SR*, and increases hyperbolically. The magnitude is affected by the characteristics of the hysteresis model (difference of energy absorbing ability).

This relation was established by experimental research [5].

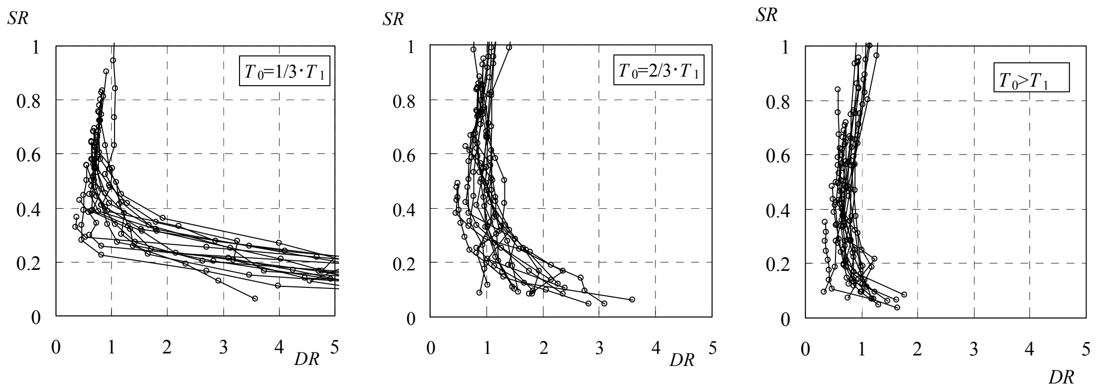


Figure 2: Normalized nonlinear displacement response

Tri-linear type hysteresis models are usually used for reinforced concrete structure members with break points of concrete cracking and main rebars yielding. The stiffness of a reinforced concrete structure degrades with 1) cracking of members, 2) yielding of members, and 3) the mechanism. For the SDOF model of a reinforced concrete structure, the model of tri-linear hysteresis with break points of the first stiffness degrading point and close to mechanism point is used.

The previous studies used bi-linear type hysteresis models. In order to apply those to reinforced concrete structures, some engineering works are required. For bending yield type structures, it was shown that displacement response can be accurately estimated by setting the initial period of the bi-linear system at $\sqrt{2}$ times in the elasticity period as mean period in the elastic period and secant modulus period at yielding point. This paper aims to extend the results obtained using bi-linear hysteresis models to the system with tri-linear hysteresis models. The Takeda tri-linear type hysteresis model [6] is used. Parameters are energy absorption ability, break points of models, yielding stiffness degrading, and type of ground motion. The following are the main considerations in this study:

1) The difference of the relation between *DR* and *SR* by energy absorption ability in case of *SR*+*TR*<1.
2) The difference of the equivalent elastic response value by the difference in the equivalent period for evaluating the tri-linear model as an equivalent bi-linear model.

SDOF ANALYSIS

The hysteresis model used for the analysis is the Takeda tri-linear type model shown in Figure 3. The stiffness after yield was 0.001 times the elastic stiffness. The parameters used in this study are summarized in Table 1.
Using this hysteresis model, 252,000 types of SDOF analysis were carried out for the 20 earthquake ground motions shown in Table 2. The damping was assumed to be 5% to the initial period and proportional for current tangent stiffness.

Calculated results were classified by the dimensionless parameters defined in Table 1. The relation of *SR* and DR are plotted for all of the calculated results of the 20 ground motions for each TR, FR, KR, and β. The value of the elastic response displacement for the non-dimensionalization of DR was taken as the 5%

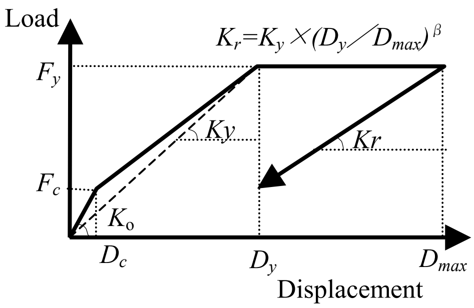


Figure 3: Trilinear Model

damping smoothed response spectrum value.

TABLE 1 PARAMETERS IN ANALYSES

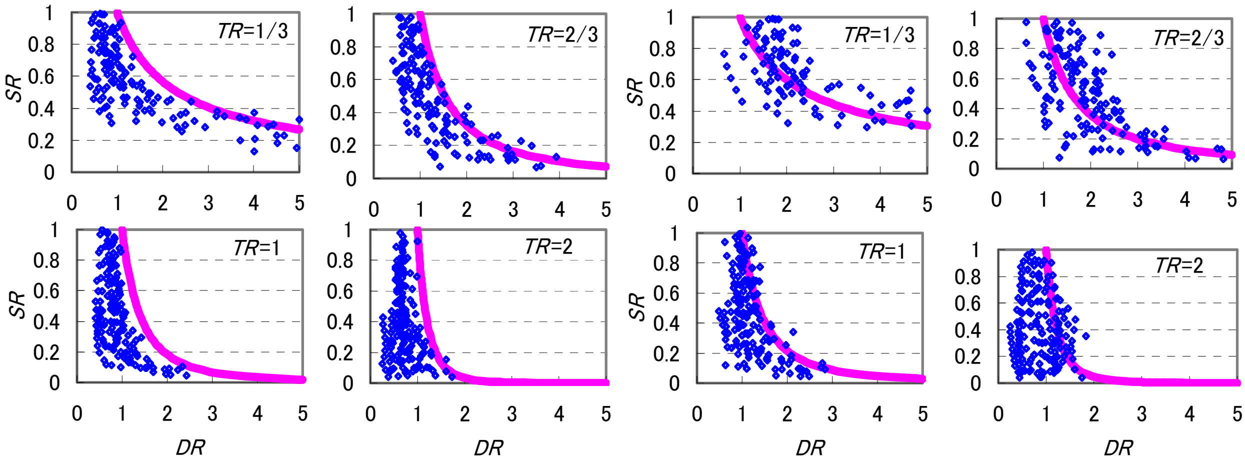
Ratio of break point in force, FR	Cracking force, Fc / Yielding Force, Fy	0.2～0.8 (0.1)	7types
Ratio of stiffness, KR	Yielding stiffness, Ky / Elastic stiffness, Ko	0.2～0.6 (0.1)	5 types
Ratio of strength, SR	Yielding strength / 5%damping elastic resistant response	0.1～1 (0.1)	10 types
Ratio of period, TR	Initial period, To / Boundary period shown in fig. 1, T_1	1/3、2/3、1、2、3、and 5sec	6 types
Unloading parameter shown in Fig. 3, β		0～1 (0.2)	6 types
Ground motions		—	20 types
Ratio of displacement, DR	Maximum displacement response / Smoothed elastic displacement response spectral value with 5% dumping factor	—	—

（DR is not a parameter of analysis. This will be used to evaluate the results）

TABLE 2 GROUND MOTIONS

No.	Ground motion	Maximum values			T_1	No.	Ground motion	Maximum values			T_1
		Acc. (mm sec^{-2})	Vel. (mm sec^{-1})	Displ. (mm)	(sec)			Acc. (mm sec^{-2})	Vel. (mm sec^{-1})	Displ. (mm)	(sec)
1	EL Centro NS	3417	335	109	0.57	11	Tho30-1FL EW	2026	276	91	0.54
2	EL Centro EW	2101	369	198	0.77	12	Castaic EW	3107	163	26	0.36
3	Taft NS	1527	157	67	0.53	13	Managua NS	3175	295	67	0.38
4	Taft EW	1760	177	92	0.51	14	Santa Barbara EW	1284	188	52	1.15
5	Tokyo 101 NS	740	76	44	0.70	15	ATS	2526	372	395	0.63
6	Sendai 501 NS	575	35	19	0.29	16	YPT	3222	880	1484	1.77
7	Sendai 501 EW	475	38	21	0.40	17	Kobe EW	6175	754	179	0.81
8	Hachinohe NS	2250	341	114	0.74	18	Kobe NS	8182	904	199	0.83
9	Hachinohe EW	1829	358	133	0.93	19	Chichi T129NS	6107	515	1521	0.48
10	Tho30-1FL NS	2582	362	145	0.98	20	Chichi T129EW	9829	724	2704	0.39

As examples, Figure 4a shows the case of FR=0.5, KR=0.5 and β=0.4, which are typical values for a reinforced concrete building. Figure 4b shows the case of FR=0.2, KR=0.2 and β=0.8 as a case with small Fc, large yield stiffness degradation and small energy absorption ability. In each graph of every TR, DR tends to increase with SR similarly regardless of the type of ground motion. In Figure 4a, DR becomes less than 1.0 approximately when TR > 1.0 and SR > 0.2. The constant displacement response rule is satisfied. However, when TR < 1, DR increases hyperbolically with decrease of SR. In Figure 4b, DR generally



(a) *FR*=0.5、*KR*＝0.5、β=0.4 (b) *FR*=0.2、*KR*＝0.2、β=0.8

Figure 4: *SR-DR* relation and hyperbolic approximated curve

increases hyperbolically with decrease of *SR* and is over 1.0. The effect of these factors on *DR* are examined on the basis of the 1,260 plotted graphs of the *SR-DR* relation by the combination of *FR*, *KR*, β and *TR*.

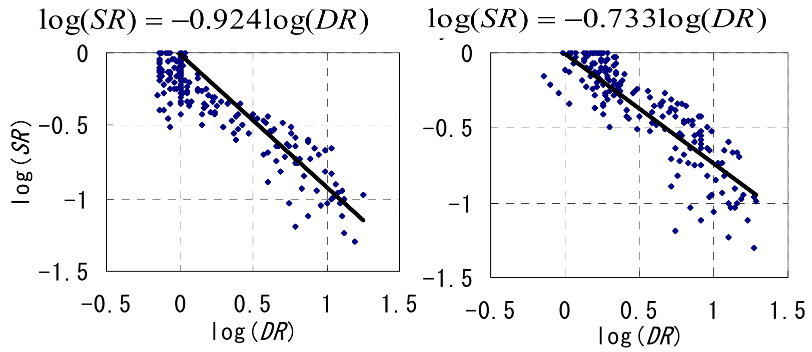
ESTIMATION FORMULA

The results were analyzed as follows:

1. The *SR-DR* relation is approximated by the hyperbola $DR = 1/SR^{1/x}$ for each graph,
2. The Relationship between $1/x$ and *FR*, *KR*, β is examined by regression analysis.
3. The Equivalent elastic displacement response value is revised to give the response at the equivalent period.

Hyperbolic approximation

The relation of *SR-DR* is approximated by the hyperbola $DR = 1/SR^{1/x}$ for each of the 1,260 graphs using the least squares method. First, the relation is plotted by taking the logarithm of *DR* and *SR* as shown in Figure 5. Coefficient x is calculated for the inclination of the linear approximation of those plots. This equation is assumed to pass through (*DR*,*SR*) = (1,1), which is the origin in the logarithm graph, because *SR*=1 means elastic response and so the displacement response should be equal to the elastic response value (*DR*=1). In the least squares method, values of *DR* of less than 0.7 are ignored to be on the safe side. Examples of the calculated line and its equation are shown in Figure 5.



(a) *FR*=0.5、*KR*＝0.5、β=0.4 (b) *FR*=0.2、*KR*＝0.2、β=0.8

Figure 5: *SR-DR* relation in the logarithm and approximated line

The relationship between calculated coefficient x and *FR*, *KR*, β for each *TR* is shown in Figure 6. In case of *TR*>1.0, *DR* becomes less than 1.0 as shown in Figure 4, and coefficient x in the equation $DR = 1/SR^{1/x}$ is more than 2 as shown in Figure 6, then the difference in the relation of *SR* and *DR* becomes very small.

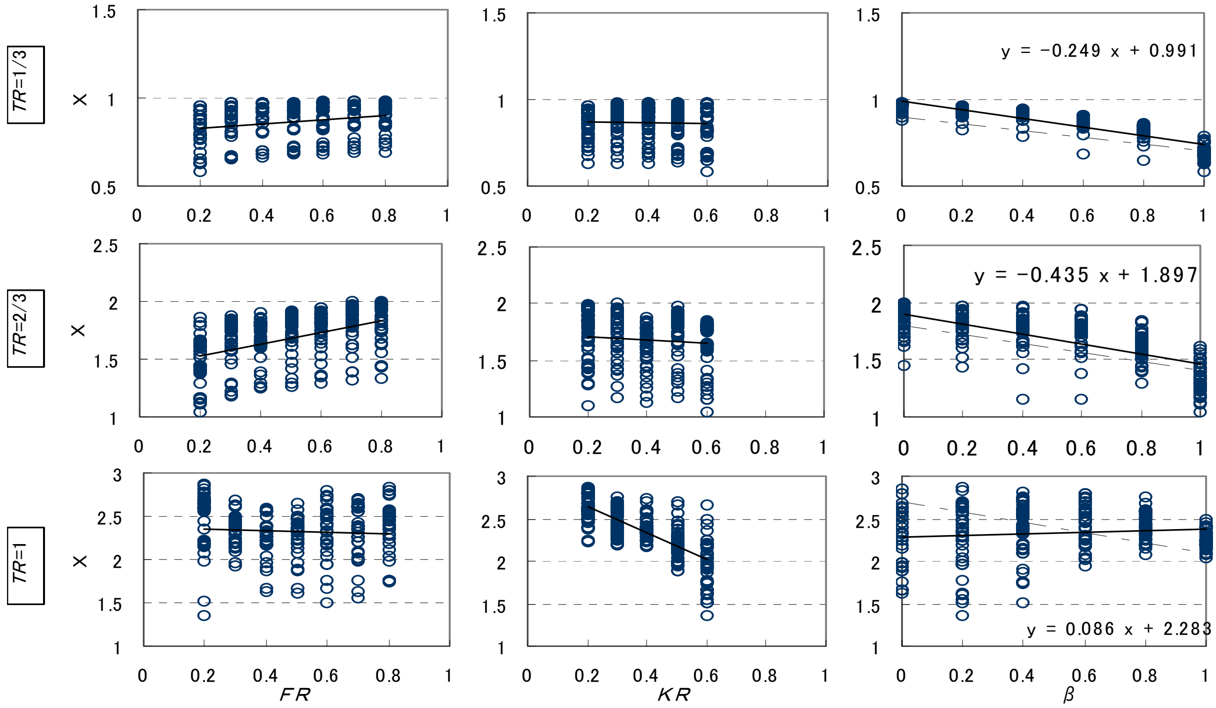


Figure 6: Relationship between coefficient x and FR, KR, β for each TR

Accordingly, this paper considers only the case $TR \leqq 1$ for investigation of coefficient x. The approximate line calculated by the least squares method is also plotted in Figure 6. Coefficient x increases with increasing FR and decreases with increasing β. From the results of multiple regression analysis, TR and β have a large t value, and both are adopted in the following simplified equation.

$$x = 3(0.9 - 0.2\beta)TR \tag{1}$$

The correlation between x values calculated by equation (1) and original data is shown in Figure 7. The coefficient of determination R^2 is 0.87. R^2 becomes 0.93 when the data of $x \geqq 2.5$ was ignored because of their little effect in the estimated formula, and the gradient of the correlation line becomes 1.0. Thus, coefficient x can be estimated by equation (1) with sufficient accuracy for practical use.

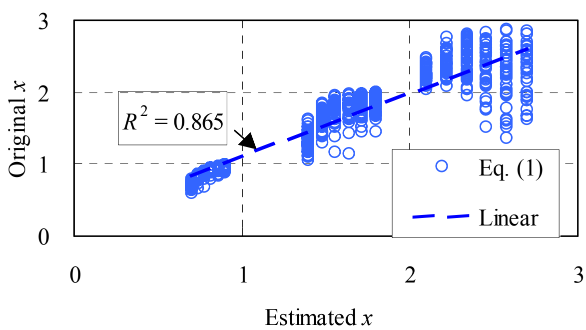


Figure 7: Accuracy of coefficient x calculated by Eq. (1)

Using these results, the following equation is obtained for predicting DR.

$$DR = \frac{1}{SR^{\frac{1}{3(0.9-0.2\beta)TR}}} \tag{2}$$

The displacement response estimated by equation (2) is shown in Figure 4. The estimation is good and on the safe side in case (a) which is for common reinforced concrete structures. In case (b), however, the evaluation is not on the safe side, especially in case of TR<1. The reason is that the initial period of the system for the tri-linear model is less representative, and some modification such as equivalent period is required.

Correction of the approximated curve

The approximation equation is revised as follows.

$$DR = \frac{\alpha}{SR^{\frac{1}{3(0.9-0.2\beta)TR}}} \tag{3}$$

The value of α is determined so as to envelope roughly the calculated results for the 1,260 graphs as shown in Figure 8. Each thin line is calculated by equation (2). The thick lines in the figures are revised to envelope the calculated results.

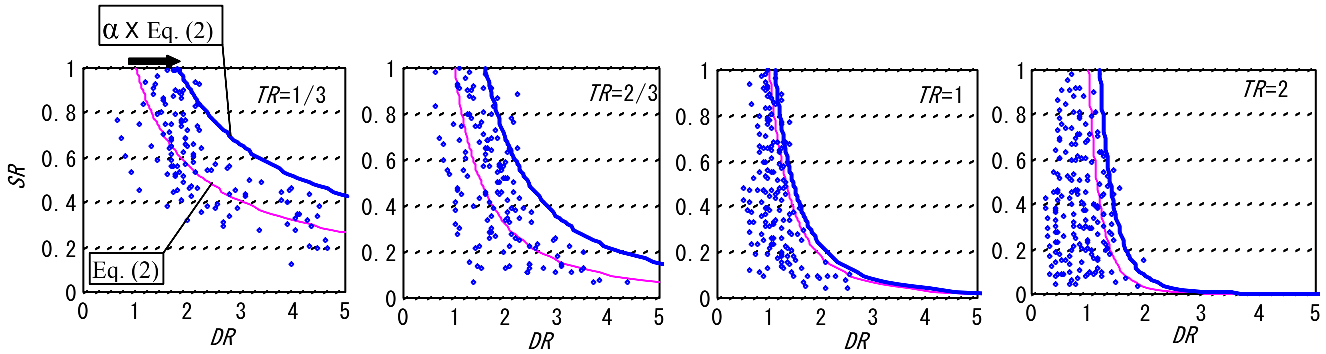


Figure 8: Example of correction by α（*FR*=0.2、*KR*＝0.2、*β*=0.4）

From the multiple regression analysis for coefficient α to *FR*, *KR* and *β*, coefficient of determination R^2 becomes about 0.75 at *TR*=1/3 and *TR*=2/3 in the relation between α and $\sqrt{FR+KR}$. Thus, linear regression between α and $\sqrt{FR+KR}$ is possible.

Figure 9 plots α on the vertical axis and $\sqrt{FR+KR}$ on the horizontal axis for each *TR*. In all figures, α is equal to 1.0 when $\sqrt{FR+KR}$ is more than 1.0, and becomes an approximately linear relation when $\sqrt{FR+KR}$ is less than 1.0. The linear regression is shown in the figure for $\sqrt{FR+KR}$ <1.0. The value of the inclination of the linear regression line shown in Figure 9 is larger than the value shown in Table 5. The relation between the inclination of the linear regression line γ and *TR* is shown in Figure 10(a). The value of γ is constant when *TR* is more than 1.0, and decreases linearly with increasing *TR*. Figure 10(b) shows the

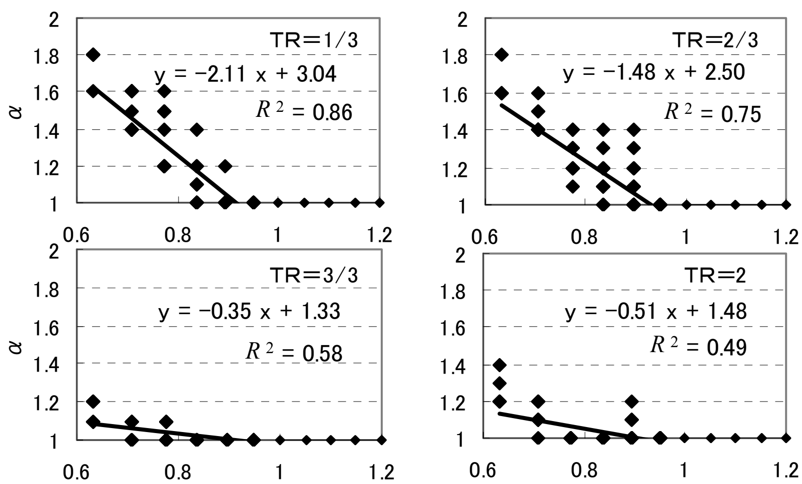
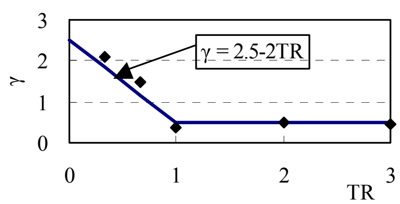
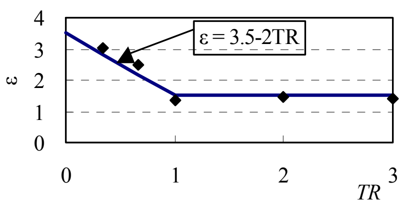


Figure 9: Relation between α and $\sqrt{FR+KR}$



(a) Relation between γ and *TR*



(b) Relation between ε and *TR*

Figure 10: Relation between γ, ε and *TR*

value of α at $\sqrt{FR+KR}$ =0 (let this value be ε). The value of ε is also constant when TR is more than 1.0, and decreases linearly with increasing TR.

The relationship between γ and period ratio TR is approximated by the following equation as shown in Figure 10(a).

$$TR<1 \qquad \gamma=2.5-2TR$$
$$TR\geqq 1 \qquad \gamma=0.5 \tag{4}$$

The relationship between ε and period ratio TR is approximated by the following equation as shown in Figure 10(b).

$$TR<1 \qquad \varepsilon=3.5-2TR, \qquad\qquad TR\geqq 1 \qquad \varepsilon=1.5 \tag{5}$$

Using equations (4) and (5), correction factor α is given by the following equation as a function of FR, KR and TR in case of $\sqrt{FR+KR}$ <1.

$$TR<1 \qquad \alpha=3.5-2TR-(2.5-2TR)\sqrt{FR+KR}$$
$$TR\geqq 1 \qquad \alpha=1.5-0.5\sqrt{FR+KR} \tag{6}$$

The thick lines in the Figure 8 are the values calculated by equation (3) using the value of α calculated by equation (6). It becomes approximately an evaluation on the safe side.
From the above, the following equation to estimate displacement response was obtained using each parameter shown in Table 1.

$$\sqrt{FR+KR}\geqq 1 \qquad DR=\frac{1}{SR^{\frac{1}{3(0.9\text{-}0.2\beta)TR}}}$$

$$\sqrt{FR+KR}<1 \qquad TR\leq 1 \qquad DR=\frac{3.5-2TR-(2.5-2TR)\sqrt{FR+KR}}{SR^{1/\{3(0.9-0.2\beta)TR\}}}$$
$$TR>1 \qquad DR=\frac{1.5-0.5\sqrt{FR+KR}}{SR^{1/\{3(0.9-0.2\beta)TR\}}} \tag{7}$$

Evaluation of displacement response estimated formula

The values estimated by equation (7) and the response analysis results are compared in Figure 11 for five kinds of ground motion of different frequency response characteristics. Here, β=1 (the origin oriented model) was ignored. For the case in which the value estimated by equation (7) exceeded the value of the equal displacement region of the response spectrum, the value at the equal displacement region was taken. These figures show that equation (7) gives estimation values on the safe side. Some estimated values are more than double the calculated values. The estimation formula presented here has been set to produce an

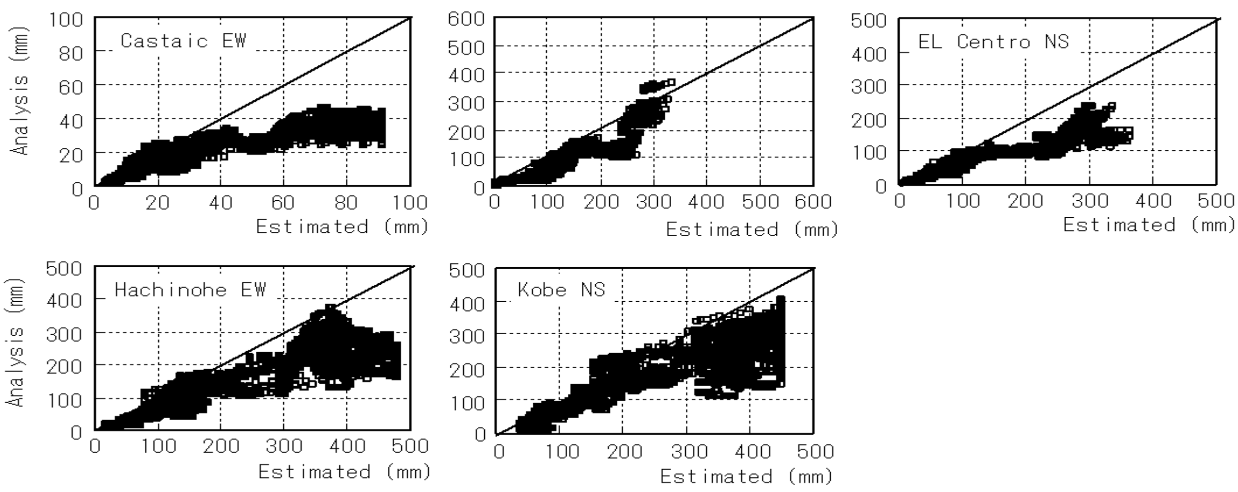


Figure 11: Comparison of estimated values and response analysis results

evaluation on the safe side. One reason is considered that the response of the system is in the valley of the response spectrum, and it may not be appropriate to use the values in the valley in the design. Another reason is that the estimated values at $DR< 1$ have all been estimated as $DR=1$.

CONCLUSION

This study carried out parametric analysis of the SDOF model with the Takeda tri-linear hysteresis model to estimate the displacement response of reinforced concrete structures. The parameters used in this study were load ratio *FR* which is the ratio of crack load to yield point load, stiffness ratio *KR* which is the ratio of secant modulus of stiffness at yield point to initial stiffness, strength ratio *SR* which is the ratio of the shear strength of the system to elastic resistant response, period ratio *TR* which is the ratio of initial period to boundary period between constant acceleration spectrum region and constant velocity spectrum region, return rigidity lowering rate β of the Takeda model, and type of ground motions. The main conclusions obtained in this paper are as follows.

1. The ratio of elasto-plastic displacement response to elastic response displacement (displacement response ratio *DR*) for a SDOF model using Takeda tri-linear hysteresis has a hyperbola relation to strength ratio *SR* regardless of the type of ground motion, when it is classified by the dimensionless parameters of *FR*, *KR*, *TR* and β.
2. Coefficient x, when the *SR*-*DR* relation is approximated by the hyperbola $DR= 1/SR^{1/x}$, is a function of *TR* and β. The rate of increase of *DR* with decreasing *SR* becomes large with decreasing *TR* and increasing β.
3. When the total of *FR* and *KR* is less than 1.0, *DR* becomes larger than the value of the hyperbola relation of *SR*. This magnitude is given by the linear relation of $\sqrt{FR+KR}$ and *TR*.
4. Nonlinear displacement response can be estimated by equation (7) on the safe side.

The response analysis program of the one-mass system used in this study was Otani SDF [7]. The author acknowledges Professor Otani of the University of Tokyo for permitting use of this program.

References

1. Qi, X. & J.P. Moehle (1991), *Displacement design approach for reinforced concrete structures subjected to earthquakes*, Report No. UCB/EERC-91/02, Earthquake Engineering Research Center, University of California
2. Shimazaki, K & A. Wada (1993), *Seismic drift of reinforced concrete structures*, Journal of Structural and Construction Engineering, AIJ, No. 444, Feb., pp. 95-104 (In Japanese).
3. Shimazaki, K. & M.A. Sozen (1984), *Seismic Drift of Reinforced Concrete Structures*, Hazama Technical Research Report, pp.145-166
4. Shimazaki, K. (1988). *Strong ground motion drift and base shear coefficient for R/C structures*, Proceedings of 9th WCEE, V pp.165-170
5. Bonacci, J. F. (1989). *Experiments to study seismic drift of reinforced concrete structures*, Ph.D Thesis, University of Illinois
6. Takeda, T., M.A. Sozen & N.N. Nelson (1970), *Reinforced concrete response to simulated earthquakes*. Journal of Structural Division, ASCE, Vol. 96, No. STc2, pp 2557～2573
7. Otani, S. (1981), *Hysteresis Models of Reinforced Concrete for Earthquake Response Analysis*, Journal of the Faculty of Engineering, pp.125-159, The University of Tokyo